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# Plane Symmetric Inflationary Cosmological Model with Thick Domain Wall in Brans-Dicke Theory of Gravitation

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***Abstract*** *We discuss the field equations of Brans-Dicke theory for a plane symmetric inflationary cosmological model in the presence of thick domain wall. We assume that the expansion scalar ($\theta$) is proportional to the shear scalar ($\sigma$) and also the power law ansatz for scalar field ($\phi$). The physical and geometrical behaviour of the resulting model is discussed through different parameters.*

***Keywords*** *Cosmology, Brans-Dicke theory, Scalar field, Domain wall.*

## 1 Introduction

The Brans-Dicke (BD) theory of gravity [1] has been extensively studied by many researchers for the last five decades in different physical contexts.This theory is consisting of a massless scalar field $\phi$ and a dimensionless constant $\omega$ describing the strength of the coupling between $\phi$ and the matter [1]. In this theory the gravity is mediated by a scalar field $\phi$ in addition to the usual metric tenson $g_{ij}$ present in Einstein's general theory of relativity. According to Mach's principle the long range scalar field $\phi$ is generated by the whole matter in the universe and has the dimension of the inverse of the gravitational constant G [2]. The work of Singh and Rai [3] gives a detailed Survey of Brans-Dicke cosmological models discussed by several authors. A revised model of the inflationary universe under the framework of Brans-dicke theory is investigated by Mathiazhagan and Johri [4]. Singh and Singh [5] have investigated a cosmological model in Brans-dicke theory by considering cosmological constant as a function of scalar field $\phi$. Obregon and Pimental [6] presented exact cosmological models with particle creation taking BD scalar field $\phi$ as a linear function of time. Berman [7] considered special law of variation of Hubble's parameter in involutionary models with perfect fluid as material source, which leads to a constant value of deceleration parameter. Ready et al. [8] studied an axially symmetric Bianchi type-I Cosmological model with negative constant deceleration parameter with the help of special law of variation of Hubble's parameter proposed by Berman [7]. Johari and Desikan [9] studied Brans-dicke cosmological models with constant deceleration parameter in the presence of creation of matter particles. Zeyauddin and Ram [10] presented two categories of exact solutions of the field equations of Brans-dicke theory for a spatially homogeneous and anisotropic Bianchi type-V cosmological models by applying the law of variation of Hubble's parameter. Venkateswarlu and Satish [11] have investigated LRS bianchi type-I inflationary string cosmological model in BD theory of gravitation. Singh and Bishi [12] presented a bulk viscous cosmological model in Brans-dicke theory with new form of time varying deceleration parameter. Ozer and Delice [13] studied gravitational waves in Brans- Dicke theory with a cosmological constant. Recently, Bamba et al. [14] have studied gravitational decoupling of anisotropic stars in the Brans-Dicke theory.

There are many open questions in theoretical and experimental cosmologies, which are still under consideration by cosmologists and need to be answered. One of the challenging problem in cosmology is the domain wall problem. Domains walls are formed when the universe undergoes a series of phase transitions with discrete symmetry being spontaneously broken (Vilenkin [15,16]). After the symmetry breaking different regions of the universe can be settling into different parts of the Vacuum with domain walls forming boundaries between their regions. The light domain walls of large thickness may have produced during the late time phase transitions such as those occurring after the decoupling of the matter and radiations (Hill et al. [17], Reddy [18]). A lot of work has been done on thick domain walls in Lyra geometry by Rahamann [19, 20]. Reddy and Rao [21] studied axially symmetric domain walls in Lyra geometry. Pawar et al. [22] studied bulk viscous fluid plane symmetric string dust magnetized cosmological model in general relativity. Katore et al. [23] have studied domain wall cosmological models with deceleration parameter in modified theory of gravitation. Katore and Hatkar [24] investigated Bianchi type III and Kantowski-Sachs domain wall cosmological models in $f$ (R,T) theory of gravitation. Recently, Rao et al. [25] have studied dynamics of cosmological model with domain walls and massive scalar fields in $f$ (R, T) gravity.

Motivated by the above observations, we consider plane symmetric inflationary cosmological model in Brans-Dicke theory of gravitation in the presence of thick domain wall. The outline of the paper is as follows: The model and the field equations are presented in Section 2. In Section 3, we deal with an exact solution of the field equations with thick domain wall. In Section 4, we describe some physical and geometrical properties of the model. The conclusions of the study are given in section 5.

## 2 Model and Field Equations

We consider the plane symmetric space-time of the form

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2, \tag{1}$$

where $A$ and $B$ are functions of $t$ only.

Brans-Dicke [1] field equations for combined scalar and tensor fields are

$$G_{ij} = -\frac{8\pi}{\phi} T_{ij} - \frac{\omega}{\phi^2}(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij}\phi_{,k}\phi^{,k}) - \frac{1}{\phi}(\phi_{;ij} - g_{ij}\Box\phi), \tag{2}$$

and
$$\Box\phi = \phi^{;k}_{;k} = \frac{8\pi}{3+2\omega} T, \tag{3}$$

where
$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R. \tag{4}$$

Here $G_{ij}$ is the Einstein tensor, $T_{ij}$ is the energy momentum tensor of the matter, $R_{ij}$ is the Ricci tensor, $\phi$ is the Brans-Dicke scalar field and $\omega$ is the dimensionless coupling constant. Also comma (,) and semi-colon (;) denote partial and covariant differentiation, respectively.

The stress-energy components in comoving co-ordinates for the domain wall under consideration here are given by

$$T_1^1 = T_2^2 = T_4^4 = \rho, \text{ and } T_3^3 = -p, \tag{5}$$

where $\rho$ is the energy density of the wall, which is again equal to the tension along $x$ and $y$ directions in the plane of the wall and $p$ is the pressure along $z$ direction.

The field equations (2) and (3) for the metric (1), in view of equation (5) are given as

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\frac{8\pi\rho}{\phi} - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{A}}{A}\frac{\dot{\phi}}{\phi} - \frac{1}{\phi}\Box\phi, \tag{6}$$

$$\frac{2\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 = \frac{-8\pi p}{\phi} - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{B}}{B}\frac{\dot{\phi}}{\phi} - \frac{1}{\phi}\Box\phi, \tag{7}$$

$$\frac{2\dot{A}\dot{B}}{AB} + \left(\frac{\dot{A}}{A}\right)^2 = \frac{-8\pi\rho}{\phi} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\ddot{\phi}}{\phi} - \frac{1}{\phi}\Box\phi, \tag{8}$$

$$\Box\phi = \ddot{\phi} + \left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{\phi} = \frac{8\pi}{3+2\omega}(3\rho - p), \tag{9}$$

where a dot (.) denotes differentiation with respect to time $t$.

We define the average scale factor $a$, the volume scalar $V$ and the generalized mean Hubble's parameter $H$ for the space-time (1) as

$$a = \left(A^2 B\right)^{\frac{1}{3}}, \tag{10}$$

$$V = a^3 = A^2 B, \tag{11}$$

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right), \tag{12}$$

where $H_1 = H_2 = \frac{\dot{A}}{A}$ and $H_3 = \frac{\dot{B}}{B}$ are the directional Hubble's parameters in the directions of $x$, $y$ and $z$ respectively.

From equations (10) and (12), we obtain

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B}\right). \tag{13}$$

The Kinematical quantities such as the expansion scalar θ, the shear scalar σ and the anisotropy parameter $A_m$ are defined as

$$\theta = \left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B}\right), \tag{14}$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right), \tag{15}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{16}$$

The deceleration parameter $q$ in a cosmological model is defined as

$$q = -\frac{a\,\ddot{a}}{\dot{a}^2}. \tag{17}$$

## 3 Solution of the Field Equations

The field equations (6) - (9) are a system of four equations with five unknown parameters $A$, $B$, $p$, $\rho$ and $\phi$. Therefore, we need more relations to find the determinate solution of the system. So any one quantity may be chosen independently to solve the system of equations. Since the field equations contain $A$ and $B$ and their derivatives, so without any loss of generality, we shall assume that the *BD* scalar field $\phi$ is some power of the average scale factor $a$. Thus, following Johri and Desikan [9], we assume a power-law relation between the average scale factor $a$ and the scalar field $\phi$ of the form

$$\phi = b a^{\alpha}, \tag{18}$$

where $\alpha$ is any integer and $b$ is the constant of proportionality.

For a spatially homogeneous metric, the normal congruence to homogeneous expansion implies that $\sigma/\theta = n$, where $n$ is a constant. This condition leads to

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=n\left(\frac{2\dot{A}}{A}+\frac{\dot{B}}{B}\right), \tag{19}$$

which yields to

$$\frac{\dot{A}}{A}=m\frac{\dot{B}}{B}, \tag{20}$$

where $m=(1+n\sqrt{3})/(1-2n\sqrt{3})$. The above equation, after integration, reduces to

$$A = lB^{m}, \tag{21}$$

where $l$ is an integrating constant.

From equations (10), (18) and (21), we get

$$\phi = \beta B^{(2m+1)\alpha/3}, \tag{22}$$

where $\beta = bl^{2\alpha/3}$. From equations (6) and (8), we obtain

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}-\frac{\dot{A}\dot{B}}{AB}-\left(\frac{\dot{A}}{A}\right)^{2}=-\omega\left(\frac{\dot{\phi}}{\phi}\right)^{2}+\frac{\dot{A}}{A}\frac{\dot{\phi}}{\phi}-\frac{\ddot{\phi}}{\phi}\ . \tag{23}$$

Using equations (21) and (22) in equation (23), we get

$$\frac{\ddot{B}}{B}+\left(\frac{(\omega-1)(2m+1)^{2}\alpha^{2}+3(1-m)(2m+1)\alpha-18m}{3(2m+1)\alpha+9(m+1)}\right)\frac{\dot{B}^{2}}{B^{2}}=0, \tag{24}$$

which on integration gives

$$B=\left[(k+1)\ (c_1 t+c_2)\right]^{1/(k+1)}, \tag{25}$$

where $c_1$ and $c_2$ are constants of integration and

$$k=\frac{(\omega-1)(2m+1)^{2}\alpha^{2}+3(1-m)(2m+1)\alpha-18m}{3(2m+1)\alpha+9(m+1)}. \tag{26}$$

Using equation (25) in equation (21), we obtain

$$A=l[(k+1)\ (c_1 t+c_2)]^{m/(k+1)}\ . \tag{27}$$

Hence the metric (1) reduces to the form

$$ds^2 = dt^2 - l^2 [(k+1)(c_1 t + c_2)]^{2m/(k+1)} (dx^2 + dy^2) - [(k+1)(c_1 t + c_2)]^{2/(k+1)} dz^2. \qquad (28)$$

## 4 Geometrical and Physical significance

The scalar field $\phi$ in the model is given by

$$\phi = \beta[(k+1)(c_1 t + c_2)]^{(2m+1)\alpha/3(k+1)}. \qquad (29)$$

We observe that the scalar field $\phi$ is time-dependent and is an increasing function of cosmic time. Therefore, during the evolution of universe the scalar field is growing and affects the behaviour of physical parameters in the model.

The spatial volume $V$ of the model is given by

$$V = a^3 = l^2[(k+1)(c_1 t + c_2)]^{(2m+1)/(k+1)}. \qquad (30)$$

The directional scale factors $A$, $B$ and the spatial volume $V$ are increasing function of time $t$. Initially, when $t \to -c_2/c_1$, the scale factors $A$, $B$ and volume $V$ attain zero value and finally, when $t \to \infty$, they attain infinite values. This shows that the model starts evolving with zero volume and attain infinite volume at final stage. The model has a point type singularity at $t = -c_2/c_1$.

The directional Hubble's parameters $H_i$ ($i$ = 1, 2, 3) and the generalized mean Hubble's parameter $H$ are given by

$$H_1 = H_2 = \frac{m c_1}{(k+1)\,(c_1 t + c_2)}, \qquad (31)$$

$$H_3 = \frac{c_1}{(k+1)\,(c_1 t + c_2)}, \qquad (32)$$

$$H = \frac{(2m+1)\,c_1}{3(k+1)\,(c_1 t + c_2)}. \qquad (33)$$

From the above equations, we observe that $H_1$, $H_2$, $H_3$ and $H$ are decreasing functions of time $t$. Hence our model is not a steady-state model.

The deceleration parameter ($q$) of the model is obtained as

$$q = -\left[1 - \frac{3(k+1)}{(2m+1)}\right]. \qquad (34)$$

We note that $q < 0$, $q = 0$ and $q > 0$, respectively indicate the phases of accelerated expansion, uniform expansion and decelerated expansion of the universe. Here the deceleration parameter $q$ is found to be negative provided $k < 2(m-1)/3$. Hence the present model represents an accelerating phase of the expanding universe. Recent observations like SNe Ia [26] and CMB anisotropy [27] confirmed that the present universe is accelerating and the value of deceleration parameter $q$ lies somewhere in the range $-1 \le q \le 0$.

The expansion scalar θ, shear scalar σ and the anisotropy parameter $A_m$ take the form

$$\theta = \frac{(2m+1)\,c_1}{(k+1)\,(c_1\,t + c_2)}, \tag{35}$$

$$\sigma = \frac{(m-1)c_1}{\sqrt{3}\,(k+1)\,(c_1\,t + c_2)}, \tag{36}$$

$$A_m = 2\left(\frac{m-1}{2m+1}\right)^2. \tag{37}$$

We observe that expansion scalar θ and shear scalar σ are decreasing function of time which have infinite value at $t = -\, c_2/c_1$, but tend to zero in late-time evolution. The anisotropy parameter is constant, which shows that the nature of the model is always anisotropic throughout the evolution. Also $\lim_{t\to\infty} \frac{\sigma}{\theta} =$ const., which shows that the shear scalar does not tend to zero faster than the expansion scalar and hence the model has anisotropic behaviour.

The energy density $\rho$ and the pressure $p$ for the model are given by

$$\rho = \left(\frac{(\omega\alpha - 6)\,(2m+1)^2\alpha - 18m\,(m+2)}{144\pi\,(k+1)^2}\right)\frac{c_1^2\phi}{(c_1\,t + c_2)^2}, \tag{38}$$

$$p = \left[\frac{18m\,(2k - 3m + 2) - (2m+1)\,\alpha\,\{(2m+1)\,(\omega+2)\,\alpha + 6\,(2m - k - 1)\}}{144\pi(k+1)^2}\right]\frac{c_1^2\phi}{(c_1 t + c_2)^2}, \tag{39}$$

where $\phi$ is given by equation (29).

From the above equations, it is observed that energy density $\rho$ and pressure $p$ are decreasing functions of time. Initially, when $t = -\, c_2/c_1$, both $\rho$ and $p$ are infinitely large and tend to zero as $t \to \infty$. The presence of the BD scalar field $\phi$ in the expressions of the physical parameters $\rho$ and $p$ is playing the important role and affect the behaviour of these parameters. Since the scalar field $\phi$ is an increasing function of time $t$ and hence its presence slows down the rate of decrease of energy density $\rho$ and pressure $p$ at late time-evolution of the universe.

## 5 Conclusion

In this paper, we have presented plane symmetric inflationary cosmological model with thick domain wall in Brans-Dicke theory of gravitation. It is observed that at the initial epoch $t = -c_2/c_1$, the physical quantities for the model (28) like expansion scalar ($\theta$), shear scalar ($\sigma$), Hubble parameter ($H$), energy density ($\rho$) and pressure ($p$) diverge. Thus our model starts with big-bang at $t = -c_2/c_1$ and goes on expanding until it comes out to rest at $t = \infty$. The initial singularity in the model is the point type. We observe that $\frac{\sigma}{\theta}$ is constant, the model does not approach isotropy at any time. Our model is in accelerating phase which is consistent to the recent observations. It is also observed that the scalar field $\phi$ is playing an important role in slowing down the rate of decrease of energy density $\rho$ and pressure $p$ in the model at late-time evolution. We have found a new solution for inflation that deserves attention.

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